The Prisoner's Dilemma: A Mathematical Analysis

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I. Introduction

a. Background

Game theory broadly refers to the theory describing and analyzing situations in which the decision of one entity and the decision of an opposing or competing entity result in a number of predetermined outcomes. The prisoner's dilemma is a subset of game theory and is simplified to give each of two parties two decisions—resulting in 2*2=4 possible outcomes. The original prisoner's dilemma was described and developed by employees of the RAND Corporation[1], a global policy think tank, and is described as follows:

> Two suspects are arrested. They are separated by the police, and each is given the opportunity to testify against the other. If one suspect testifies against the other (hereafter termed "defects") while the other remains silent (hereafter termed "joins"), the defector goes free and the joiner is sentenced to 10 years in prison. If both suspects join (remain silent), each will serve a six month sentence. If both suspects testify (both defect), each will serve five years in prison. What should each prisoner do?

b. Equilibrium

This situation will be analyzed mathematically later, but the constants involved in the analysis need to be introduced here. Therefore, let

> **A = Defection reward**—the sentence given to the prisoner who defects while the other joins. In the above situation, this reward is defined as going free.
>
> **B = Joining reward**—the sentence given to each prisoner if both join. Above, B is defined as 6 months of jail time.
>
> **C = Defection penalty**—the sentence given to each prisoner if both defect. Above, C is defined as 5 years of jail time.
>
> **D = Joining penalty**—the sentence given to the prisoner who joins while the other defects. Above, D is defined as 10 years of jail time.

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[1] Prisoner's Dilemma (Stanford Encyclopedia of Philosophy). (1997, September 4). *Stanford Encyclopedia of Philosophy*. http://plato.stanford.edu/entries/prisoner-dilemma/ (accessed May 03, 2011).

Looking at the situation logically, one may be tempted to join. If both prisoners remain silent, each will only serve 6 months—a much shorter sentence than one of 5 or 10 years. However, if the prisoner is interested only in serving the minimum sentence, defection is always the preferable option. If the other prisoner joins, defection results in freedom; if the other prisoner defects, defection avoids a 10-year sentence. To see this more clearly, we will redefine A, B, C, and D as potential rewards instead of punishments. Consider their values as being points. Obtaining maximum points is as desirable as avoiding prison, and the prisoners are now described as players. We reassign their values to be

$A = 7$ $B = 5$ $C = 2$ $D = 1$

Choosing to defect ensures that a player will receive either A or C. Choosing to join ensures B or D. The points can be chosen arbitrarily, but, as long as $A>B>C>D$, it stands to reason that defection is always the better option, regardless of the other player's decision. The game, with this inequality stipulated, yields what is known as a "Nash Equilibrium," named after the famous mathematician John Forbes Nash, Jr. (yes, the one from *A Beautiful Mind*). He describes an equilibrium as existing in a game in which both players are aware of the reward structure, each is knowledgeable of his or her opponent's options, and, when each makes a rational decision considering those options, there is always one decision that is better than the other. In the prisoner's dilemma in which $A>B>C>D$, both players defecting is a Nash Equilibrium. This will be proven mathematically later, as will the fact that A must exceed B for the game to be playable.

c. Non-equilibrium

Suppose instead that the joining penalty is not as severe as the defection penalty. That is, keeping silent as the other prisoner testifies does not carry as steep a prison sentence as does each prisoner testifying against the other. In terms of points, $A>B>D>C$. For instance,

$A = 7$ $B = 5$ $C = 1$ $D = 2$

A player who plans to defect must now face the realization that defecting is a worse option than joining if his or her opponent also plans to defect. The Nash Equilibrium is thereby disrupted: the "correct" move depends upon the move of the opponent, rather than simply the logic of the situation. If A>B>D>C, the prisoner's dilemma is a non-equilibrium game. The correct decision, then, can be considered to be some function of the probability that the opponent defects. Suppose Bob and Steve happen to love anything having to do with math and, as a result of this unyielding love, agree to play the game (each seeking to maximize his own points). If Steve considers Bob to be prone to defect, how should Steve act? If Bob is a trustworthy soul who is likely to join, should Steve automatically choose to defect? And are there turning points at which the strategy changes? This will all be analyzed mathematically later.

d. Applications and Survey

The thinkers at RAND did not develop the prisoner's dilemma as an esoteric thought exercise. Rather, there existed then and now exist a number of decisions in many facets of humanity that can be simplified to a prisoner's dilemma. These include decisions in politics, social policy, law enforcement (obviously), sports, and economics. Two in particular are quite applicable today: whether a government should sign a climate change treaty that may potentially harm its economy but help the environment and whether an athlete should take performance enhancing drugs at the risk of bodily harm. It is immediately apparent that the cost and benefit of each decision depends a great deal on the behavior of the other entities, be they governments or athletes. We will analyze these situations later, accompanied by the results of a survey of hundreds who were asked to decide as though they were the diplomat and the baseball player.

e. Investigation

Lastly, I performed an in-person, real life, and real time test of the prisoner's dilemma with both equilibrium and non-equilibrium values. The "reward" was a certain number of Silly Bandz (which, as you may know, are all the craze among kids these days. If you have had the misfortune of never encountering Silly Bandz, they are essentially rubber bands that each form

into different shapes—including animals, fairies, sports gear, and so on). Players were presented with the four outcomes based upon their defecting or joining and were told to make their decisions. Those decisions will be analyzed not just for what they were (join or defect) but also for how the players reported arriving at those decisions.

Before we get to all that, though, I should confirm that a mathematical analysis of the equilibrium and non-equilibrium assessments made above match those obtained logically. This will be done by developing an equation for an expected outcome as a function of the probability of each player's defecting. It's time for Bob and Steve to square off:

II. Mathematics

We now define the following variables:

x = the probability that Bob defects
y = the probability that Steve defects
z = Steve's expected value

We also need to recall the following constants:
A = the defection reward (score for a player defecting when opponent joins)
B = the joining reward (score for a player when both join)
C = the defection punishment (score for a player when both defect)
D = the joining punishment (score for a player joining when opponent defects)

We can then generate an expected value function z, (of x and y) from Steve's perspective of:

z = (probability that both defect)(defection punishment) + (probability that Bob defects and Steve joins)(joining punishment) + (probability that Steve defects and Bob joins)(defection reward) + (probability that both join)(joining reward)

Or in terms of the variables and constants:

$$z = xy(C) + x(1-y)(D) + y(1-x)(A) + (1-x)(1-y)(B) \quad (1)$$

By foiling and distributing (1), we obtain

$$z = Cxy + Dx - Dxy + Ay - Axy + B - Bx - By + Bxy \quad (2)$$

By investigating (2) in various scenarios, we can determine some criteria for the game to have or not have a Nash Equilibrium. First, assume Steve chooses to defect, freezing the value of y at 1. Equation (2) reduces to

$$z = Cx + Dx - Dx + A - Ax + B - Bx - B + Bx \quad (3)$$

Which, upon simplification, reduces further to

$$z = Cx - Ax + A \quad (4)$$

Equation (4) demonstrates two desired properties. First, if Bob defects (x=1), Steve's result, z, is simply C, the defection punishment. Secondly, if Bob joins (x=0), Steve's result is simply A, the defection reward.

If we instead assume Steve chooses to join, freezing the value of y at 0, equation (2) reduces to

$$z = Dx + B - Bx \quad (5)$$

with no further simplification possible. Analogous to the result of equation (4), equation (5) reduces to z = B, the joining reward, if Bob also joins and reduces to z = D, the joining punishment, if Bob defects.

The logic described in the introduction argues that there exists a Nash Equilibrium of both players *always* choosing to defect when A > B > C > D. It would be edifying to see if equations

(4) and (5), which give Steve's expected value upon defecting and joining, lead to the same conclusion. In order for Steve to even consider joining, his expected value upon joining, equation (5), must exceed the expected value upon defecting, equation (4). Mathematically,

$$Dx + B - Bx > Cx - Ax + A \tag{6}$$

Recall that x is the probability that Bob defects. Rearranging (6) gives

$$Dx - Bx + Ax - Cx > A - B \tag{7}$$

Factoring out an x on the left side and dividing by the constant terms gives

$$x > \frac{A - B}{D - C + A - B} \tag{8}$$

To restate, in order for Steve to consider joining, his expected value upon joining must exceed his expected value upon defecting. Both expected values depend upon x, Bob's probability of defecting. Equation (8) is the resulting inequality, stating that Steve should join if the probability that Bob defects exceeds a certain value. I will return to this shortly. First, equation (8) is deceptively revealing about the nature of the prisoner's dilemma. The first mathematical observation to note is that x, representing a probability, must be between 0 and 1 (probabilities can, of course, equal 0 or 1, but I am assuming that no decision has been officially made). For any fraction to fall into this domain restriction, the denominator must exceed the numerator. That is,

$$D - C + A - B > A - B \tag{9}$$

Canceling out A - B on each side, equation (9) reduces to

$$D - C > 0$$

or

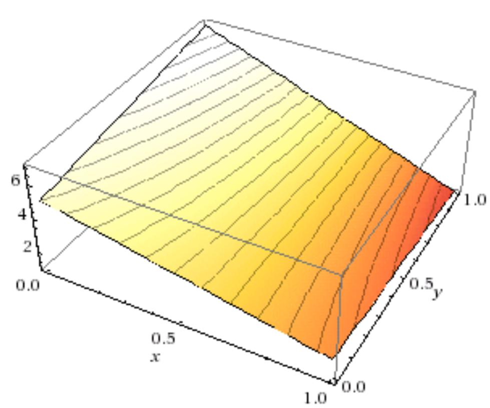
$$D > C \tag{10}$$

This seemingly minor result has major implications. In order for Steve to even consider joining, the probability the Bob defects reduces to equation (8). In order for the probability that Bob defects to actually be a probability, the value of D, the joining punishment, must exceed the value of C, the defection punishment. In other words, being betrayed must have a more attractive outcome that mutual defection. The logic in the introduction led to the same conclusion: that to establish a non-equilibrium, there must be a circumstance in which joining is more attractive than defecting. The simple inequality of (10) confirms that making D exceed C is just such a method. The other logical method, making B exceed A, wreaks havoc on the inequality of (8). The numerator will be negative, as will the denominator, unless D again exceeds C by a larger margin than B exceeds A. However, assigning values in such a manner causes a new equilibrium, this side in favor of joining. To wit: If the reward for both participants' joining exceeds the reward for successful betrayal, and the punishment for unsuccessful betrayal exceeds the punishment for being betrayed, a player's best interest is solely to join.

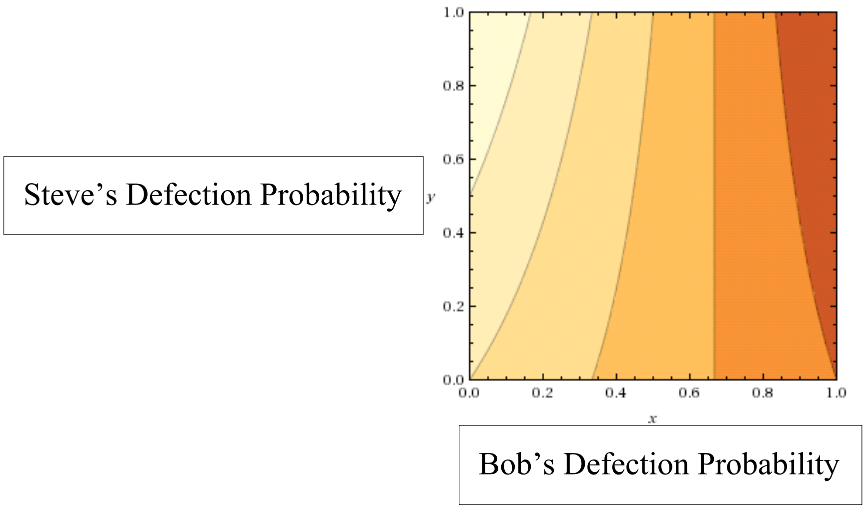
The conclusion is that the only way to avoid a Nash Equilibrium in the prisoner's dilemma is to hold the following restriction on values:

$$A > B > D > C \qquad (11)$$

Which agrees with the logic of the introduction. At this point we have manipulated equation (2) in a variety of definitive scenarios. Let's investigate it in its original form, as the dominating equation describing Steve's expected value, z, in terms of his own defection probability, y, and Bob's defection probability, z. This equation seems convoluted, so it may be useful to examine it graphically. For the constants, we will assign A = 7, B = 5, C = 1, and D=2, which yields a non-equilibrium situation. The three-dimensional graph appears as follows:



It certainly appears that the greatest expected value for Steve (depicted on the vertical axis, z) arises when he is most likely to defect (high value for y) and Bob is most likely to join (low value for x). But does that mean that for Steve, defection is always the best strategy? Examine a two-dimensional contour diagram of the same function:



The light color to the upper left represents the highest expected value for Steve, while the dark color to the right represents the lowest. The contour lines connect values of equal expected value. For instance, Steve's expected value when his own defection probability is 1.0 and Bob's is 0.5 is equal to his expected value when his defection probability is 0.0 and Bob's is about 0.35.

Notice that if Steve wants to reap the greatest reward, he must, in all likelihood, defect (y>0.6). The same strategy, though, exposes Steve to a wide swath of potentially low rewards, depicted by the darker sections to the right. Indeed, a visual inspection based on geometric probability dictates that the approximate area of the darkest swath when y>0.6 exceeds the area of the darkest swath when y<0.6. In other words, if Steve knows nothing about Bob's strategy, but Steve wants to avoid risking the lowest reward, he should consider joining over defecting. If Steve is quite sure Bob will join (say, x<0.2), Steve's clear strategy is to defect (y>0.6). If Steve is quite sure Bob will defect (x>0.6), his only respite is to join (y<0.4) to avoid the lowest reward.

Graphically, it seems the best non-equilibrium strategy for Steve is to make the opposite decision that Bob does. Is this algebraically confirmed? To further investigate the strategy in the only non-equilibrium situation, we revisit equation (8). This result dictated that Steve should join if and only if the probability that Bob would defect exceeded a rational number:

$$x > \frac{A - B}{D - C + A - B}$$

We will now introduce two new values to ease the analysis of this inequality. If we let H = A - B and K = D - C, (8) simplifies to

$$x > \frac{H}{H + K} \quad (12)$$

For purposes of clarity, we can describe H as the "defection enticement," as it represents how greatly the reward for successful betrayal exceeds the reward for mutual joining. Likewise K could be considered the "joining enticement," as it represents how much milder the punishment for being betrayed is than the punishment for mutual defection. Again, x is the probability that Bob defects. Imagine the following values are set for A, B, C, and D, and thus H and K:

A = the defection reward = 14
B = the joining reward = 4
C = the defection punishment = 2
D = the joining punishment = 3
thus
H = 10
K = 1
and

$$\frac{H}{H + K} = \frac{10}{11} = 0.91(approx)$$

By this conclusion, Steve should join if the probability that Bob defects is greater than 0.91, or highly likely. But why should Steve join if the most probably outcome is that he is betrayed? To avoid earning the low value of C, the punishment for mutual defection. Even though A is high, at 14, it is extremely unlikely that Steve can earn it, considering the high probability of Bob's defection. Note, though, that if the probability that Bob defects is not great (suppose he is a "trustworthy" player), Steve should almost certainly defect. In other words, these values for the rewards and punishments algebraically promote a strategy of defection. If instead, the values are assigned as:

A = the defection reward = 8
B = the joining reward = 7
C = the defection punishment = 1
D = the joining punishment = 5
thus
H = 1
K = 4
and

$$\frac{H}{H+K} = \frac{1}{5} = 0.2$$

Now, Steve's strategy is that he should join if the probability that Bob defects is greater than 0.2. In other words, Steve should join unless he is very confident that Bob will join as well. This conclusion is driven by the relatively low value of successful betrayal over successful joining. Finally, to restate the conclusion from (11), let:

A = the defection reward = 10
B = the joining reward = 7
C = the defection punishment = 3
D = the joining punishment = 1
thus
H = 3

K = -2

and

$$\frac{H}{H+K} = \frac{3}{1} = 3$$

which is an impossible probability, and thus nonsensical.

The conclusion from the first two applications of (12) is that the only successful strategy in a non-equilibrium prisoner's dilemma is to make the opposite decision of one's partner. The three-dimensional graphs generated by equation (2) ascribe to this conclusion as well. But does this agree with real-life applications of the prisoner's dilemma whose outcome values suggest the non-equilibrium requirement of (11)?

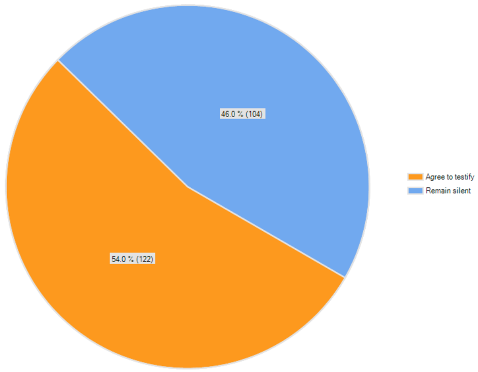
III. Survey Results

a. Classic prisoner's dilemma

At this point, we have demonstrated both logically and mathematically that a player's best strategy in the classic prisoner's dilemma, with A>B>C>D and a Nash Equilibrium decision, is to defect (agree to testify). Naturally, I wondered what decision real people, who were uninitiated into the logic or algebra of the dilemma, would make if put in a similar situation. So, I asked 226 people to anonymously answer the following question:

> Suppose you are a bank robber (fun!). You and your colleague are both arrested and charged with armed robbery. You are immediately placed in two separate and noise-proof rooms. Police then present you with the following choice: You, handcuffed to table, can either testify against your colleague or remain silent (note: your colleague is given the same choice). You have four possible outcomes: 1. You testify against your colleague and your colleague remains silent. As a result, you **go free**, and your colleague receives a **10-year sentence**. 2. You both remain silent. As a result, you both receive **6-month sentences**. 3. You both testify against each other. As a result, you both receive **5-year sentences.** 4. You remain silent and you colleague testifies against you. As a result, you receive a **10-year sentence** and your colleague **goes free.** In deciding, realize that your colleague is aware of the same outcomes. What do you do?

The participants were given two choices either to **agree to testify** or to **remain silent.** The results of this survey question are as follows:



As you can see, 54% of the people surveyed made the correct decision, opting to testify. Although this accounts for more than half, it was surprising that more participants did not act on the reality that defection carries the lighter prison sentence regardless of the decision made by the opponent. As a follow up to the decision, I asked participants to answer the question "what motivated your decision?" A few observations:

1. A large number of those who chose to remain silent did so because they either did not want to rat out their partner in crime or that they trusted their partner to remain silent.
2. Many who chose to remain silent used terms such as "guilt" or "conscience," suggesting that forcing their colleague into a longer prison sentence would be met with regret. I carefully chose the word "colleague" in the survey to suggest that the crime was a purely business endeavor. Nonetheless, it is interesting to note that so many participants lent humanity to the imagined scenario.
3. Most of those who chose to testify claimed to make the decision solely in order to minimize their prison sentence. It is impossible to know whether this choice was made with the knowledge (as described in the introduction) that defection was always preferential or that they were simply tempted by the opportunity to go free.
4. It was interesting that, of those who discussed their *colleague's* choice as at all pertinent, the vast majority chose to remain silent. The armchair psychologist would be free to claim that the simple act of considering another person's position promotes empathy and perhaps compassion. The mathematician would note that doing the same risks 10 years in prison.

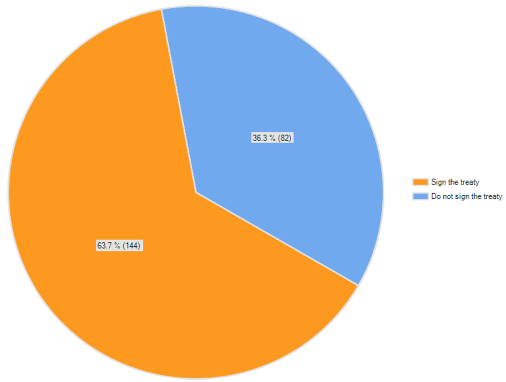
b. Climate change policy

In order to make the (mathematically) correct decision less obvious and explore a real life scenario in which two choices lead to four possible outcomes, I asked a question about a political decision relating to the economics of climate change policy. The description is intentionally meant to resemble the choices facing nations regarding the Kyoto Protocol of 1997. The United States very conspicuously chose not to ratify the treaty, citing (probably correctly) that the required decrease in carbon dioxide output would require changes to our economic structure that, at least in the short run, would have deleterious effects.[2] I chose not to use the word "Kyoto" in my question in order to avoid potential reflex decisions. The outcomes were again designed to keep the A>B>C>D inequality. The following question was asked:

> You are the president of an industrialized country. You are at a conference of nations in which the discussion is centered on taking steps to decrease fossil fuel use, thereby reversing climate change. A treaty is created that would require the signees to take steps to alter their economies in a manner that would reduce fossil fuel emissions. You, pen in hand, are faced with the following choice: You can either sign the treaty or not sign the treaty (note: the other leaders are given the same choice). You have four possible outcomes: 1. You don't sign the treaty, and most other countries do. As a result, **your country's economy booms, and the climate suffers somewhat.** 2. You sign the treaty, and most other countries do as well. As a result, **your country's economy remains stable, and the climate improves.** 3. You do not sign the treaty, nor do most countries. As a result, **your country's economy remains stable, and the climate suffers.** 4. You sign the treaty, but few other countries do. As a result, **your country's economy suffers, and the climate suffers.** In deciding, realize that the other leaders are aware of the same outcomes**.** What do you do?

[2] "Kyoto Protocol." *United Nations Framework Convention on Climate Change*. http://unfccc.int/kyoto_protocol/items/2830.php (accessed May 20, 2011).

The participants were given two choices either to **sign the treaty** or to **not sign the treaty**. The results of this survey question are as follows:



The question was designed to have a Nash Equilibrium position of not signing the treaty, as outcome 1 was meant to be preferable to outcome 2 and outcome 3 preferable to outcome 4. In this case, however, only 36.3% chose not to sign the treaty. Nearly 2/3 of the participants chose to *avoid* the potential reward of outcome 1—that their country's economy booms. Again, I asked for the motivation behind the decisions. A few notes:

1. Many who opted to sign professed a concern over the world's environment as more significant than the country's economy. An armchair political scientist would probably point out that the United States would never elect someone who professes to rank his or her priorities in that order.
2. Those who refused to sign did indeed take a mathematical angle. Of the four options, only outcome 2 results in the earth's environment's improving. The fact that 75% of the outcomes result in the climate "suffering" or "suffering somewhat" enabled these individuals to default into keeping their own economy in their interest. This, of course, implies that knowledge of many other countries' plans to sign as well would change the participants' decisions to signing the treaty.
3. Some who agreed to sign the treaty, as well as some who did not, professed their belief that the climate would not improve no matter who signed the treaty. They were right. In reality, very few of the nations who signed the Kyoto Protocol are anywhere near their stated goal in decreasing carbon emissions.

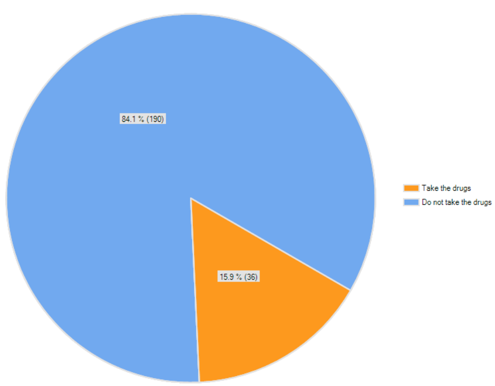
c. Drugs in baseball

The third and final survey question was designed to exhibit a non-equilibrium configuration of outcomes. That is, A>B>D>C. The premise was that using performance-enhancing drugs in baseball, such as steroids and human growth hormone, seems to offer a distinct competitive advantage at the risk of bodily harm. The advantage would be maximized if one player alone gained it. The question:

> Suppose you are a professional baseball player. Years of hard work have finally paid off. However, you suspect that many players are using performance-enhancing drugs to gain an unfair advantage. You know that using these drugs will improve your stats, resulting in a salary increase. You also know, however, that these drugs would have adverse side effects on your body. You, holding a syringe in your hand, are faced with the following choice: You can either use the drugs or not use the drugs (note: all other players in the league have the same choice). You have four possible outcomes: 1. You use the drugs, and few other players do. As a result, your **salary increases by 50%, and you obtain the side effects.** 2. You do not use the drugs nor do most of the other players. As a result, your **salary increases by 10%, and you do not obtain the side effects.** 3. You use the drugs, as do most of the other players. As a result, your **salary remains the same and you obtain the side effects.** 4. You do not use the drugs and most other players do. As a result, your **salary remains the same and you do not obtain the side effects.** In deciding, realize that the other players are aware of the same outcomes. What do you do?

The participants were given two choices either to **take the drugs** or to **not take the drugs**.

The results of this survey question are as follows:



Of the three surveys, this had the most overwhelming majority: 84.1% chose not to take the drugs. Unlike the previous scenarios, which were designed with a Nash Equilibrium in mind, this question allowed for the possibility that joining was preferable to defecting. Again, the question of motivation was posed, and a few generalizations were noted:

1. Most participants claimed to be motivated by the fact that drugs are harmful to the body and that the long-term side effects outweighed the salary increase opportunity. This was the argument put forward by former player Jose Canseco in his testimony to Congress in 2005. He noted that for a great athlete, steroids are both "unnecessary" and "harmful" while expressing heartfelt remorse for the families who suffer due to the use of performance enhancing drugs. It is interesting to note that neither he nor the majority of survey participants mentioned that it is against the rules. Perhaps right and wrong are not as significant to most as health and wealth.[3] Indeed, Canseco made upwards of $45 million as a player and millions more authoring a book exposing the use of steroids in baseball, including his own.[4]

2. Many of those who chose to take the drugs astutely noted that they did not expect many other players to do so. That, accompanied by the whopping 50% salary increase, was enough to make them choose to take the drugs. This is an instinctual confirmation of the power of equation (12), which stated that a player should only opt to join if he or she thought it likely that others would defect, and defect only if he or she thought it likely that others would join. These participants considered the risk to their health acceptable only if it would pay off with option 1 rather than option 3. In other words, Steve was thinking Bob was trustworthy fellow—the perfect opportunity for Steven to betray Bob.

3. The most recent league-wide test in Major League Baseball occurred in 2003, in which 104 of the 750 major league players tested positive for banned substances, or 13.9%.[5] This figure is remarkably close to the number of survey participants who chose to take the drugs: 15.9% out of 226.

IV: In-person Experiment Involving Silly Bandz

The final component of this investigation involved a real time and in-person test of prisoner's dilemma strategy. Rather than lack of jail time, the "reward" was Silly Bandz (see

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[3] "Quotes From Congressional Steroid Hearing." *NBC Sports*. http://nbcsports.msnbc.com/id/7223536/ (accessed May 15, 2011).

[4] "Jose Canseco Baseball Stats." *Baseball Almanac - The Official Baseball History Site*. http://www.baseball-almanac.com/players/player.php?p=cansejo01 (accessed May 15, 2011).

[5] Roberts. "Baseball's Steroid Era: News, Lists, Quotes, Timelines, Statistics." *Baseball's Steroid Era*. http://www.baseballssteroidera.com/ (accessed May 15, 2011).

introduction for brief description of these magical inventions). The experiment was performed as follows:

1. Two willing participants were identified and assured that they would win some number of Silly Bandz by participating in an experiment for a math project.
2. Each participant was told that he or she would be given the choice to join or defect and that the distribution of Silly Bandz would depend on his or her choice as well as the choice of the partner.
3. Each participant was handed 3 cards: One reading "Defect," one reading "Join," and one listing the four possible outcomes. The outcome cards read as follows:

   For the experiment in which A>B>C>D:

You	**Other Person**	**You Get**	**He/She Gets**
Defect	Joins	**7** silly bandz	**1** silly band
Join	Joins	**5** silly bandz	**5** silly bandz
Defect	Defects	**2** silly bandz	**2** silly bandz
Join	Defects	**1** silly band	**7** silly bandz

   For the experiment in which A>B>D>C:

You	**Other Person**	**You Get**	**He/She Gets**
Defect	Joins	**7** silly bandz	**3** silly bandz
Join	Joins	**4** silly bandz	**4** silly bandz
Defect	Defects	**2** silly bandz	**2** silly bandz
Join	Defects	**3** silly bandz	**7** silly bandz

Note: any individual pair participated in only one of the two types of experimental trials.

4. Once participants had made their decisions, but before they revealed them, they were asked to face each other and explain their motivations for their decisions.
5. Participants simultaneously revealed their choices.
6. SILLY BANDZ WERE REWARDED! ☺

Results of this experiment:

For trials in which A>B>C>D (Nash Equilibrium exists with defection):

- 50% of participants joined and 50% of participants defected
- 80% of trials included a defection, of which 75% were successful defections. (Note: successful implies that the other party joined).
- 80% of trials included a join, of which only 25% were successful. (Note: successful implies that both parties joined).

- The average payout in Silly Bandz for a joiner was 2.6 while the average payout for a defector was 5.0

For trials in which A>B>D>C (no equilibrium exists):

- 65% of participants joined and 35% of participants defected.
- 60% of trials included a defection of which 83% were successful defections.
- 90% of trails included a join of which only 44% were successful.
- The average payout in Silly Bandz for a joiner was 3.5 while the average payout for a defector was 5.6

As the experimenter with knowledge of the underlying logic, I expected more defections in the equilibrium trials. The fact that half of the participants joined in these trials demonstrates that the joining participants failed to recognize that defection was always preferential. The surprise with which the defection was met along with comments made prior to the revelation of choices suggest a great deal of trust between participants. The 75% successful defection rate demonstrates that this trust is illogical.

In the non-equilibrium trials, I expected strategy to be more important. Using the numbers from the non-equilibrium trials, equation (12) yields x>.75 meaning that a player should join if and only if the probability of his or her partner's defecting is 75% or higher. It seems clear from participants' anecdotal comments that the expectation was that most participants would join. The correct strategy, then, would be to defect. It was no surprise when the application of this strategy paid off.

V. Conclusion

The prisoner's dilemma, at its core, is a hypothetical. It implies a simplification of thought that, from sections III and IV, does not seem commonplace. The Nash Equilibrium solution to the A>B>C>D prisoner's dilemma requires a complete lack of consideration for the faceless "colleague" described in the survey. The Kyoto Protocol survey scenario requires a disregard for the environment of the planet in the name of the national economy, if the Nash Equilibrium solution is chosen. The Silly Bandz Experiment's Nash Equilibrium assumes that a participant is most interested in his or her own gain, at the potential loss of a competitor's. High percentages of respondents demonstrated an unwillingness to make these concessions. The irony of this apparent compassion is that the few participants in each component of the study who were

willing to defect received disproportionate benefits for doing so. In other words, testifying prisoners will be going free while their trusting colleagues will serve the maximum prison sentence; the economies of few nations will boom while the planet will continue to suffer; the defector's wrist will be covered in Silly Bandz (and will, in turn, lead to wild popularity) while the competitor will have to live with only one. Will those who chose to join in both real and imagined scenarios do so again when their "reward" is consistently paltry or nonexistent compared to the benefits won by defectors?

The survey had three two-choice questions, giving 8 possible answer choices for all three. A full 33.2% of participants chose to join in all three scenarios, more than double what random chance would dictate. This is not as simple as voting straight party ticket at an election—these honest souls had to choose to remain silent, sign a treaty, and refuse to take drugs—three actions unrelated except by the shared risk of being victimized by defectors. How many chose to defect in all circumstances? A mere 6.6% testified, did not sign the treaty, and took the drugs. Individual perspective would vary in labeling one group as somehow "better" than another, but in mathematics, the strategy favors the defecting minority.

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